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Boston University

School of Education

Thesis

A COMPARISON OF THE RESULTS OF TWO METHODS OF TEACHING FRACTIONS

Submitted by

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(B.S. in Ed., Boston University

School of Education, 1938)

(In Partial Fulfillment of the Requirements

for the degree of Master of Education)

1941

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CHAPTER I. INTRODUCTION

Methods of teaching are constantly changing. Changes are being made in the content material of all courses of study. Ballard[1] in the preface to his book refers to the subject matter taught and says, "The only conclusion I can come to is that textbooks are too good for the pupils. The arithmetical fare we offer them is ill-suited to their digestions; it is either too rich or too much. What we need is a simpler diet, a diet which they can manifestly turn into sound bone and muscles, a diet for which a keen appetite is the best sauce. From what I can discover the appetite, at present, is none too good."

In the early days most subjects were taught deductively[2]. A definite rule was given, then much practice in following the rule. Not very much thought was given to having a child understand the "why" of a situation. If an answer could be given in "parrot form", it was considered excellent. Colburn[3] is the father of the inductive method in arithmetic. In arithmetic it seemed to fit the situation better than the deductive method. To see practical uses for the various rules and then to generalize seemed to be a more logical way to proceed. A wise use of both methods will probably bring best results. However, it is easier to begin inductively and later, when maturity requires it, use the deductive methods.

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[1] Ballard, Philip B. Teaching the Essentials of Arithmetic. University of London Press, London, 1928, preface X.

[2] Monroe, Walter. "Development of Arithmetic as a School Subject" Department of Interior, Bureau of Education Bulletin 1917, No. 10: 93, 125.

[3] Monroe, Ibid 85.

Ballard[4], in Chapter III explains that even adults do not know the reasons for many of the rules they follow. "People use the rules of arithmetic as arbitrary modes of procedure: they do not indulge in the luxury of explaining them." At one lecture which he gave before a group, a bishop seemed very attentive, particularly when he described and explained the various procedures in subtraction. After the lecture the bishop spoke to Mr. Ballard and explained that he had never known the reasons for doing the things he did when subtracting and seemed pleased to know that there were definite reasons to explain the situation. Reasons for doing what we do, in arithmetic, should be explained as early as possible. It is not necessary that a child go through the whole reasoning scheme each time he uses the process, but at some time, it should be explained, that he may appreciate and see through the whole scheme. Deferring the beginning of the study of arithmetic facilitates the development of the child's judgment and understanding. It should probably be deferred until grade three. Understanding all this, we still cling to much of the traditional and close our eyes to the facts which would help to change all this.

There are two articles in the "School Science and Mathematics" magazine trying to give the "why" of some of our fraction processes. Dickey[5] describes, giving various examples and tells how this process, the division of fractions, may be demonstrated by cutting up pieces of paper. He says, "In

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[4] Ballard, Op. Cit.

[5] Dickey, John W.: "Why Invert and Multiply." School Science and Mathematics. 36: 299-302 Mar. 1936.

conclusion, the answer to the why of the inversion of the divisor and of the change of operational sign in the division of fractions, is, that it renders the denominator common and non-effective and places the numerator in an indicated division." Spear[6] has a method of working with fractions called the "unity method". This he develops even into or through the teaching of algebra, to give reasons for doing the various tricks necessary to get the proper results.

Guiler[7] attempted to find a method of improving ability to work with fractions. A standard test was given, and the results analyzed. Later, after working for improvement, this test, in a different form, was given again. Among his conclusions he states the following: "Pupils in each grade varied in mastery of the field and specific uses of fractions; wide individual differences in the amount of improvement; and marked improvement in ability to use fractions may be expected from a remedial program which discovers individual weaknesses and then provides instruction and practice definitely suited to the pupil's needs." From the reading of these articles and many others, it is safe to assume that further consideration needs to be given curriculum material. Worry over fractions disappears if the modern viewpoint is considered.

An effort to find the reasons for the common errors found in working with fractions was made by Olive A. Kee[8] in her master's thesis. After analyzing each paper and listing for each process, the reasons for the failures to get the correct

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[6] Spear, Joseph: "Mathematics - To Reason Not Just To Do." School Science and Mathematics. 38: 402-410 Apr. 1938.

[7] Guiler, Walter S.: "Improving Ability in Fractions." Mathematics Teacher. 29: 232-240 May 1936.

[8] Kee, Olive A.: "An Analysis of Errors in Operations With Common Fractions" Master's thesis 1927.

answers, she gives the following outstanding errors:

General confusion of rules	39.6%
Failure to reduce	14.2%
Substitution of another process	30.6%
Errors in changing to common denominator	4.4%
Right process started, definite error traced	11.2%

This study was made in 1927, several years before Dalrymple[9] worked on his study of fractions used most commonly in business and life. General studies by Wilson, Wise, Woody, and Charters looked toward the same goal. This story[10] has been adequately told elsewhere.

Dalrymple studied the fractions used most commonly by Sears Roebuck & Company in its various departments, S. S. Pierce Company, United Drug Company, Jordan Marsh Company, Statler Hotel, Copley Plaza Hotel, financial pages of the Boston Transcript, and the First National Bank, all of Boston, Massachusetts. These companies are varied enough to include all industries. In his conclusions he states that previous studies found fractions over sixteenths could be profitably eliminated. This study concludes that fifths, sixths, sevenths, ninths, tenths, elevenths, thirteenths, fourteenths, and fifteenths seldom or never occur; even eighths and twelfths occur only in specific situations and not in combination with other fractions. Dalrymple went on to discover to some extent what relationship might be found between units of measurements and fractions. Of the 20,641 fractions studied in the Sears

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[9] Dalrymple, C. O.: "Fractions of Business and Life." Doctor's dissertation, 1933.

[10] Wilson, Guy M.: *What Arithmetic Shall We Teach*. Riverside Press, Boston, 1926 84-104.

Roebuck & Company, 1/2, 1/3, 1/4, 3/4, 1/8, 3/8, 5/8, 7/8 comprised 94.229% of all. The most important units of measurement or items to which fractions were applied were inch, size, number, wool, yard, foot, paint, quart, ounce, pound, horsepower, cent, carat. These represented 97.785% of all the different times such fractions may have occurred. Thus, he says, drill should be eliminated on the more difficult fractions and centered on halves, quarters, and thirds in simple situations.

Shea[11] then made a study of the fractions used in textbooks. He concludes that all textbooks "carry fractions much beyond usage, leading to waste of school time or useless and unnecessary discouragement and failure." Comparing the fractions as counted and used in these textbooks with Dalrymple's study, Shea says that the Hoyt-Peet "New Everyday Arithmetic", the best in his list, goes far beyond Dalrymple's findings.

Keeping in mind these fractions which have been found to be used to the greatest extent and the causes of common errors in working with fractions, two definite plans of attack in the teaching process may be formulated to introduce this work. The one most commonly followed is the manipulative method, the using of a textbook, not deviating very much from what it prescribes. The other plan is suggested by Dr. Wilson in an article by Wilson and Dalrymple[12] entitled "Useful Fractions". It says "indications are that this simple program of mastery in fractions can best be accomplished through an objective,

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[11] Shea, Joseph B.: "Frequency of Occurrence of Fractions in Seven Textbooks." Master's thesis 1934.

[12] Wilson and Dalrymple: "Useful Fractions" Journal of Educational Research. 29: 347-354 Jan. 1937.

non-manipulative procedure. Any further program should be entirely informational."

In this study these two methods of teaching fractions will be used; the results compared, as a group and individually, noting the kinds of errors, and in which situation they more often occur.

Since our school system, a town near Boston, uses the book "Modern School Arithmetic" by Clark, Otis, Hatton the work presented to the class will follow this as closely as possible.

CHAPTER II. METHOD OF PROCEDURE

Introduction

In doing the work for this study, fractions were taught in two different ways to two separate classes. These children were paired and grouped for analyses of their work. For this grouping the National Intelligence Tests were given. Not only the children's mental age and intelligence were considered, but also their home environments and the background of nationality and education. These were paired as equally as possible with a limited number of children.

The work followed the method presented in the textbook used in this school system, since it is from the examples in this book that the tests were made by the supervisor. Many fractions were used which, in former research work, have been found to be of little or no value. These were presented to the class as "just-for-fun work", as much of it may never be used in later life.

Manipulative Method

During the school year 1937 and 1938 fractions were taught, subjectively and manipulatively, in the regular stereotyped manner. The textbook, as to method of teaching, was followed closely and the fractions were taught as abstract numbers to be manipulated and worked with according to prescribed rules.

In order to add fractions-

1. they must first be changed to a common

denominator

2. only numerators are added
3. results are considered as to whether they should be reduced or changed to a mixed number, then reduced
4. then, the whole numbers are added
5. the composite results are obtained.

An example of this is

$$\begin{array}{rl} 6\ 1/4 & 1/4 \\ 7\ 1/2 & 2/4 \\ 4\ 3/4 & 3/4 \\ \hline 18\ 1/2 & 6/4 = 1\ 2/4 = 1\ 1/2 \end{array}$$

Answer 18 1/2

In the same manner subtraction was taught. When subtracting mixed numbers where borrowing was concerned, difficulty was always encountered. A few children had to be drilled and drilled on these facts: that, in such an example-

$$\begin{array}{rrr} 3\ 1/2 & 2/4 & 2\ 6/4 \\ -1\ 3/4 & 3/4 & 1\ 3/4 \\ \hline & & 1\ 3/4 \end{array}$$

1. change to a common denominator
2. can you subtract 3/4 from 2/4? No.
3. borrow 1 from the whole three leaving 2
4. change borrowed 1 to 4/4
5. but we already have 2/4, so 2/4 and 4/4 is 6/4
6. therefore we now have 1 3/4 to be subtracted from 2 6/4.

Steps 4 and 5 were only mastered by rote except by a certain few. They could see no reason for doing it except that the teacher said it was the proper thing, and by doing this, the correct results were obtained.

Multiplication proved to be a reversal of all rules already taught. First, the child learned to change everything to the same denominator before trying to further manipulate the numbers, now he finds it not necessary. To multiply a fraction by a fraction is the easiest of all the fraction processes. The children must learn all the various multiplications as separate steps, such as, a whole number times a fraction, a whole number times a mixed number, and so on, and a set of rules for each. The child must recognize each setup and know what rules to use when each occurs.

For many, cancellation proves to be a Waterloo. Why it should be necessary to cancel a numerator and a denominator is beyond comprehension when the two numerators or two denominators can be canceled so much more readily!

Division, being the last of the processes taught, involves more hazards than the other three processes. Should these be changed to common denominators, should they be multiplied, or must the new set of rules, of inverting before multiplying be applied? If it is necessary to invert, which must be inverted or are both inverted?

In following the manipulative type of teaching, the rules are set up and each example is first classified as to which set of rules is to be used, then these rules are applied and the results are obtained.

Objective Method

During the school year 1938 and 1939 fractions were taught

objectively and non-manipulatively. By this is meant a method of teaching whereby the child is taught how to think through each situation and is thus enabled to see through each process step more readily.

Fractions were introduced, in our school system, in the third grade. When a child learned the story of 2 x 5, 5 x 2, 2)10, 5)10, he also learned 1/5 of 10, and 1/2 of 10. He learned this for every combination. In the fourth grade, fractions were studied further. The children learned to add such combinations as 1/2 + 1/4. They also learned to find 3/4 of, 2/3 of, 4/5 of numbers.

In the fifth grade, an attempt was made, the first of the year, to give them a comprehension of what fractions are and what they mean. The first lesson was a discussion of the uses of fractions by their parents. It was found that they were used only occasionally. Then a discussion followed as to why the fifth grade should study and be able to think through the processes using fractions. As in the previous class, it was decided to study these fractions "just for fun" and if an occasional use was found for them, so much the better. Much interest was shown. The class was told that this was an experiment to see if they could reason through the fraction work easier with this new way we were going to try. They were also told that the results of their lessons were to be compared with the results obtained in the previous class which was now in the sixth grade.

Succeeding these lessons, we had several review lessons reminding the class of the things they already knew. Before studying any of the fraction processes, we had many lessons coloring in parts of pictures of fences, pies, etc., comparing the parts of each as to size, and, in as many ways as possible, giving a concrete meaning for the words-"whole number," "fraction," and "mixed number." Lessons which pleased the class a great deal and seemed to give them an insight into what fractions were, were the actual "cutting-and-comparing lessons." Demonstrations, using one apple or a bar of candy, had been made, but the children wanted to do some of this actual comparing themselves.

Two papers with twelve dittoed circles were given to each member of the class, also a few pieces of arithmetic paper. The circles were cut out and called pies. Some of them were left whole, others were cut into halves, quarters, thirds, eighths, and one was cut into sixteenths. Each child kept his own in an envelope, to refer to at any time he wished. In order to show that halves were not always the same size, we cut up the arithmetic paper in the same way. These were used as puzzles; quarters were compared with halves, eighths, sixteenths and so on.

Addition of fractions was taught in this same way. Apples, oranges, bars of candy, pies, pictures of fences, and many other articles were actually used and put together. One or two illustrations may show the way in which these were used with the

children. By demonstration, one bar of chocolate was left whole, others were broken into sections as 1/2's, 1/4's and so on. The various fractional parts were fitted together and compared with the unbroken bar of chocolate. Different fractional combinations were experimented with. The children found that sometimes they got, as a result, more than the size of one bar and sometimes less. After these class demonstrations, the children, in their spare time, took out their own pies and papers and played with them, fitting various pieces together, comparing with the uncut pieces to see what they had as a result.

Subtraction was taught in the same way. When it came to borrowing in subtraction, no one had any difficulty seeing why. When it was necessary to take 1 1/2 from 5, five circles were placed on the desk in front of each child. In order to do the subtraction one of these whole ones must be cut in halves. That made four whole ones and two halves. After subtracting, there were 3 1/2 left. The harder borrowing subtractions, as to take 1 3/4 from 2 1/2, were demonstrated by cutting up one pie into fourths, doing the subtracting, and then studying the results. The answer is 3/4.

Before going on to the multiplication lessons, I introduced the terms "finding common denominator," "reducing fractions," "changing to mixed numbers," or "changing to improper fractions." No drill was given on any of these, but each child was led to understand the meaning of each.

The multiplication process offered some difficulties. Instead of using our pies for this, we used money in half-dollars and quarters, and yard goods in halves, quarters, and eighths. These were all presented in problem form, as- "If six children were given 1/4 dollar to spend, how much would they have all together?" or- "Mother wants to make "sister dresses" for Joan and Mary. She needs 2 7/8 yards for each. How much cloth will she need to buy for the two dresses?"

Later we used our pies to find 1/2 of 1/4, 1/4 of 2/3, etc. by actually cutting the sections into pieces that made the solution possible, as we had been told, and comparing the results with a whole one, or with the parts we already knew, to see what we had. Pupils soon saw that 4ths of 3ds called for 12ths.

Division was the hardest of all to demonstrate. Such examples as- "If I had 3/4 of a pie and 6 people came to dinner, how much of a pie would each person eat?" $(3/4 \div 6 = 1/8)$ or- "If I had 3 apples and I wanted to give each person 1/2 an apple, how many people could I serve?" $(3 \div 1/2 = 6)$. The yardstick was used to figure- "If I had 2 yards of cloth and I wanted to make towels 3/8 yard long, how many could I make?" $(2 \div 3/8 = 5\ 1/3)$.

The objective group has manifested far more interest in arithmetic than any of the fifth grades for a long time.

Testing for Results to Compare

In October of both years a similar test was given. These

included finding parts of numbers and changing fractions to larger denominators. All of the examples in the various tests were taken from the textbook used in our school system. Copies of these tests will be found in the Appendix, pp. 81.

Since addition and subtraction processes had been taught by the end of February, a test on these, made up of examples given in the textbook, was given the latter part of the month. The same test was given to both groups.

In the early part of May a test was given including all the processes using fractions. These examples were all taken from a page entitled "A Fraction Review." A copy of these tests will be found in the Appendix, pp. 83.

In order to find how much each group carries over into the next grade and how quickly they relearn the fraction work, two tests were given after each class entered the sixth grade, and again in the seventh grade, one in October and one in December. These included all kinds of examples using fractions. The test given in October included some examples that might be considered less useful fractions. They are very similar in form and will be found in the Appendix, pp. 84, 85.

CHAPTER III. A COMPARISON OF RESULTS

Grouping

The grouping of these children for ability is shown in Table I. The manipulative group ranges from an I.Q. of 119 to 76 with a median I.Q. of 105, and a mean I.Q. of 103.04, while the objective group ranges in I.Q. from 120 to 73, median I.Q. 104 and mean I.Q. 102.67. The median Chronological Age (C.A.) of the manipulative group is 10 years 5 1/2 months, while the median C.A. of the objective group is 10 years 7 months. The mean C.A. of the manipulative group is 10 years 7 1/8 months, and of the objective group is 10 years 8 5/8 months. The median Mental Age (M.A.) of the manipulative group is 10 years 10 1/2 months, the mean Mental Age 10 years 10 7/8 months, while the median M.A. of the objective group is 11 years and the mean M.A. is 10 years 11 7/12 months.

Results

Table II, listing the children in each group in the same order as in Table I, shows the scores in the first testing in October. The median of both groups was 100, while the mean of the manipulative group was 92.42% and the mean of the objective group was 94.75%. The standard deviation of the manipulative group was 10.52 as compared with 9.23 in the objective group. In both groups, the score ranged from 100 to 60, a variation of 40 points, while 13 (54.17%) in the manipulative group had 100 and 14 (58.33%) in the objective group had 100.

Figure 1a shows the distribution of scores in the October

Table I showing the pairing of the two groups, manipulative and objective, based upon chronological age (C.A.), mental age (M.A.), and intelligence quotient (I.Q.), together with summary data on the two groups.

Manipulative				Objective			
	C.A.	M.A.	I.Q.		C.A.	M.A.	I.Q.
Marie	10- 8	12-11	119	James	10- 7	12- 9	120
Richard M	9-10	11-10	118	Carolyn	10- 3	12- 2	119
Carol C	10- 4	12- 2	116	Robert	10- 5	12- 4	118
Ingrid	10- 6	12- 3	115	Kathryn	9-11	11- 7	117
James	10- 9	12- 5	114	Donald	10- 6	12	114
Robert B	10- 2	11- 7	113	Craig	11- 2	12- 2	109
Betsy	10- 7	11-10	111	Virginia	10- 8	11- 6	108
Betty	10	11- 3	111	Mary	10- 6	11- 4	108
Barbara	10- 4	11- 3	108	Gilbert	10- 3	11	107
Marjorie	10- 2	10-10	106	Thelma	10- 3	11	107
Verne	9-10	10- 4	105	Dorothy	10-10	11- 6	106
Richard H	10- 4	10-11	105	Barbara	10- 7	11	104
Eleanor	10- 5	10- 7	101	Henry	10- 6	10-11	104
Phyllis	10- 3	10- 4	101	Phyllis	10- 2	10- 7	104
Florence	10- 7	10- 5	99	Newton	10- 3	10- 5	102
Louise	11- 2	11	99	Russell	10- 6	10- 7	101
June	10- 8	10- 4	97	Myron	11- 5	11- 5	100
Allister	11- 3	10-11	97	Herman	11	10-11	99
Robert S	10- 3	9- 8	95	Nadine	11- 2	10-11	98
Thomas	10-11	10- 4	95	John	11	10	91
Regina	10- 8	10	94	Curtis	11- 2	10	90
Kenneth	10-11	9- 8	90	Margaret	10- 9	8-11	83
Carol	11- 6	10	88	Marshall	12- 3	10	82
David	12- 2	8-11	76	Frederick	11- 2	8- 2	73

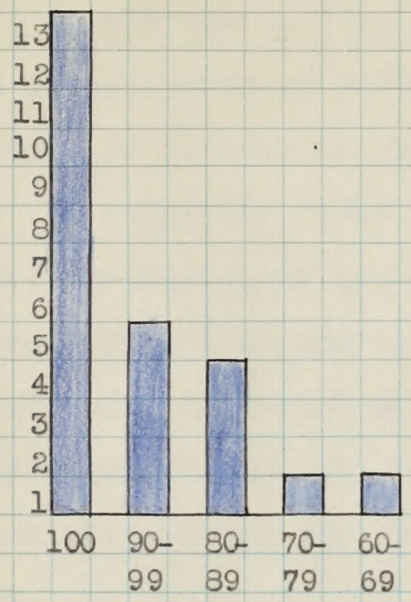
	C.A.	M.A.	I.Q.	C.A.	M.A.	I.Q.
Median	10-5½	10-10½	105	10- 7	11- 0	104
Mean	10-7 1/8	10-10 7/8	103.04	10- 8 5/8	10-11 7/12	102.67

	Manipulative	Objective
Number of boys	9	14
Number of girls	15	10
Total	24	24

Table II showing the scores of the two groups, manipulative and objective, in the first testing in October, together with summary data.

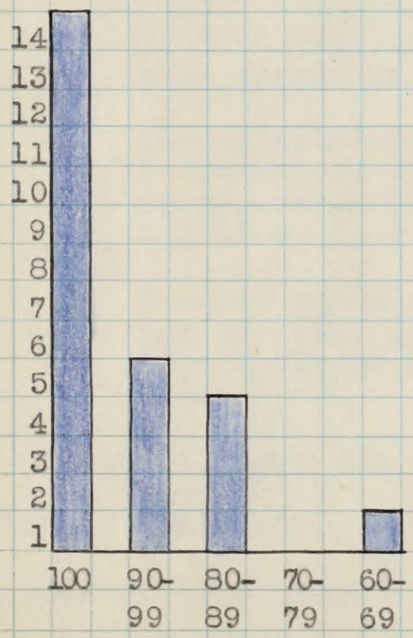
Manipulative		Objective	
Marie	100	James	100
Richard M	80	Carolyn	96
Carol C	100	Robert	90
Ingrid	90	Kathryn	100
James	100	Donald	90
Robert B	100	Craig	60
Betsy	100	Virginia	88
Betty	100	Mary	88
Barbara	90	Gilbert	100
Marjorie	100	Thelma	100
Verne	100	Dorothy	100
Richard	100	Barbara	100
Eleanor	80	Henry	98
Phyllis	99	Phyllis	100
Florence	100	Newton	100
Louise	80	Russell	80
June	60	Myron	86
Allister	90	Herman	100
Robert S	100	Nadine	100
Thomas	80	John	100
Regina	90	Curtis	100
Kenneth	79	Margaret	98
Carol P	100	Marshall	100
David	100	Frederick	100
Mean	92.42		94.75
Median	100		100
Q1	100		100
Q3	85		90
S.D.	10.52		9.23
Number of Pupils	24		24

Difference in means	2.33
Difference in S.D.	1.29
Difference in Q3	5



Mean	92.42
Median	100
Q1	100
Q3	85
% perfect	54.17
S.D.	10.52
Number of pupils	24

Figure 1a. Graphic representation of Table II, showing on October, 1937, the distribution of scores in the manipulative group, together with the summary data.



Mean	94.75
Median	100
Q1	100
Q3	90
% perfect	58.33
S.D.	9.23
Number of pupils	24

Figure 1b. Graphic representation of Table II, showing on October, 1938, the distribution of scores in the objective group, together with the summary data.

test in the manipulative group. It is read "13 made scores of 100, 5 made scores from 90 to 99," and so on. Figure 1b shows the distribution of scores in the October test in the objective group. It is read in the same way as figure 1a.

In comparing these two figures, the medians are the same, while the objective group shows a 2.33% advantage in the mean score. The first quartiles of both remain at 100, while the third quartile in the manipulative group is 85, five points lower than the 90, the third quartile of the objective group. The difference in the standard deviation of the two groups is 1.29.

Table III lists some of the examples from each group and the errors made in doing them. Those occurring most often and common to both groups are:-

> To find only one part when several parts were asked for, i.e., 2/3 of 15 is 5.
>
> Inaccuracies due to the fact that the combinations are not known well enough, as 5/6 of 24 is 15, 1/6 of 24 is 3, 5/6 of 24 is 5 x 3 or 15.
>
> Similar mistakes occurred when changing 3/4 to /12, 12 ÷ 4 equals 4, 4 x 3 is 12, 12/12; or 3/4 equals 2/8 instead of 6/8.

Table IV shows the scores in each group in the February testing. The median of the manipulative group was 80, while the mean of this group was 84.17. The median of the objective group was 90, while the mean was 85. The standard deviation (S.D.) of the manipulative group was 11.15, as compared with 16.43 in the objective group. The scores in the manipulative

Table III listing the kinds of errors found in the two groups, manipulative and objective, in the October test.

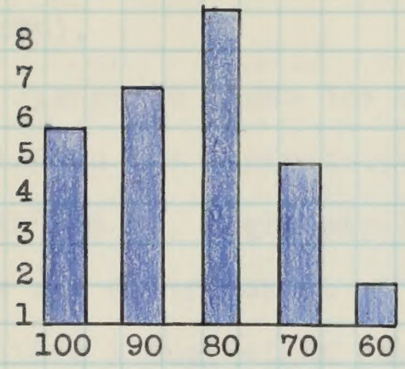
Manipulative	Objective
2/3 of 15 equals 6	5/6 of 24 equals 4
2/3 of 15 equals 5	5/6 of 24 equals 15
5/8 of 16 equals 2	3/6 of 18 equals 3
3/4 of 16 equals 4	7/8 of 32 equals 4
1/2 of 18 equals 6	2/3 of 213 equals 71
5/6 of 24 equals 4	2/5 of 25 equals 6
3/4 equals 12/12, 9/16	3/4 equals 2/8
5/6 equals 8/12, 9/12	7/8 equals 14/24
1/3 equals 5/12	

Table IV showing the scores of the two groups, manipulative and objective, in the February test, together with summary data.

Manipulative		Objective	
Marie	100	James	100
Richard M	100	Carolyn	80
Carol C	100	Robert	60
Ingrid	80	Kathryn	90
James	80	Donald	90
Robert	80	Craig	90
Betsy	90	Virginia	80
Betty	80	Mary	50
Barbara	100	Gilbert	100
Marjorie	90	Thelma	90
Verne	70	Dorothy	100
Richard H	100	Barbara	90
Eleanor	90	Henry	100
Phyllis	80	Phyllis	100
Florence	90	Newton	100
Louise	90	Russell	70
June	60	Myron	90
Allister	70	Herman	90
Robert S	80	Nadine	70
Thomas	80	John	100
Regina	70	Curtis	90
Kenneth	90	Margaret	40
Carol P	70	Marshall	100
David	80	Fred	70

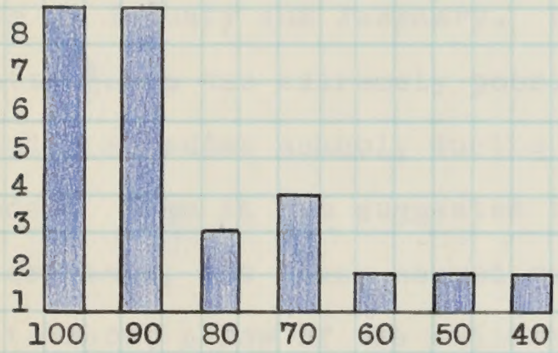
Mean	84.17	85
Median	80	90
Q1	90	100
Q3	80	75
S.D.	11.15	16.43
Number of Pupils	24	24

Difference in means	.83
Difference in medians	10
Difference in Q1	10
Difference in Q3	5
Difference in S.D.	5.28



Mean	84.17
Median	80
Q1	90
Q3	80
% perfect	20.83
S.D.	11.15
Number of pupils	24

Figure 2a. Graphic representation of Table IV, showing on the February test, the distribution of scores in the manipulative group, together with summary data.



Mean	85
Median	90
Q1	100
Q3	75
% perfect	33.33
S.D.	16.43
Number of pupils	24

Figure 2b. Graphic representation of Table IV, showing on the February test, the distribution of scores in the objective group.

group ranged from 100 to 60, a variation of 40 points, while the scores of the objective group ranged from 100 to 40, a difference of 60 points. In the manipulative group there were five 100's (20.83%) and in the objective group eight 100's (33.33%).

Figure 2a shows the distribution of scores in the February testing in the manipulative group; five pupils had 100, six pupils had 90, eight pupils had 70, one pupil had 60. Figure 2b shows the distribution of scores in the February test in the objective group; eight pupils had 100, eight had 90, two had 80, three had 70, one had 60, one 50, and one 40.

These low marks in the objective group may be explained in this way. An epidemic of grippe went through this class in the months of January and February. During three of these weeks, the attendance was extremely poor. Two of the weeks, only 16 or 17 pupils attended school; during the other week, 9 or 10 pupils attended. When it was suggested that it was time for the February test, the class seemed very ready for it, but as the results show, a few of the children had not understood the work well enough to get passing grades.

When comparing figures 2a and 2b, the median, 90, of the objective group was 10 points higher than the median, 80, of the manipulative group, while the objective group shows a .83 advantage in the mean score. The first quartile of the objective group is 100, 10 points higher than the first quartile, 90, in the manipulative group, while the third quartile of the

objective group, 75, is five points lower than the third quartile, 80, in the manipulative group. The difference in the standard deviation of the two groups is 5.28.

Table V lists some of the errors common to both tests in order of occurrence. More mistakes were found in the subtraction examples than in the addition. The mistake occurring most often in this test was the failure to reduce the answer to lowest terms. This mistake should be of little concern. The errors due to lack of drill in the addition and subtraction facts are the most important and should be given the most attention.

The final test for Grade 5 was given the latter part of May. Table VI shows the scores in each group, together with the summary data. The median of the manipulative group was 82.5, while the mean of this group was 74.88. The median of the objective group was 91.5, while the mean was 87.33. The standard deviation (S.D.) of the manipulative group was 21.56, as compared with 11.80 in the objective group. The scores in the manipulative group ranged from 100 to 17, a variation of 83 points, while the scores in the objective group ranged from 100 to 61, a difference of 39 points. In the manipulative group there were no 100's, and in the objective group six 100's (25%).

Figure 3a shows the distribution of scores in the May testing in the manipulative group; three pupils had scores between 95 and 100, six pupils had scores between 90 and 94,

Table V listing the kinds of errors found in the two groups, manipulative and objective, in the February test.

Addition

1. Failure to reduce answers
2. Incorrect addition of numbers (Did not know number facts)
3. Such mistakes as 2/5 equals 2/10 instead of 4/10
4. Failure to carry whole number

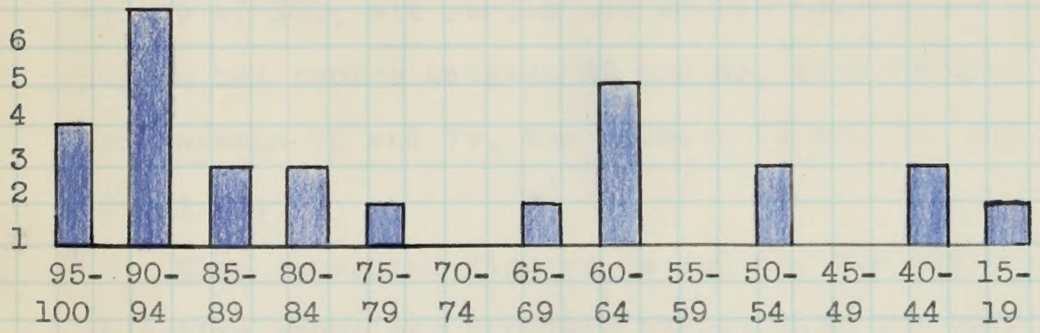
Subtraction

1. Failure to reduce answers
2. Failure to borrow, just bringing down fractions, as 5 - 4 2/3 equals 1 2/3
3. Not changing to common denominator properly before borrowing, as 1/2 and 4/4 equals 5/4
4. Adding fractions and subtracting whole numbers
5. Subtraction number facts not known

Table VI showing the scores of the two groups, manipulative and objective, in the May test, together with summary data.

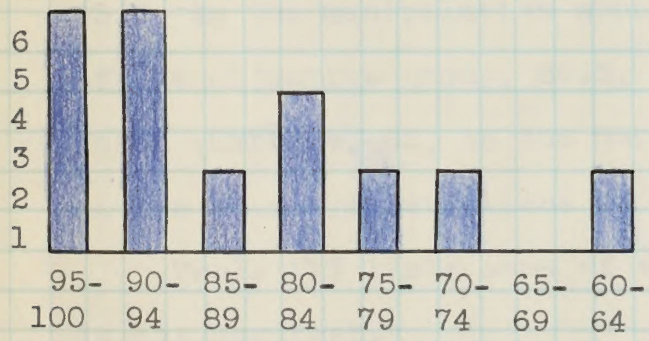
Manipulative		Objective	
Marie	97	James	100
Richard M	94	Carolyn	100
Carol C	93	Robert	78
Ingrid	94	Kathryn	100
James	98	Donald	94
Robert B	43	Craig	100
Betsy	52	Virginia	100
Betty	60	Mary	94
Barbara	82	Gilbert	94
Marjorie	98	Thelma	89
Verne	64	Dorothy	100
Richard H	94	Barbara	94
Eleanor	68	Henry	83
Phyllis	88	Phyllis	72
Florence	78	Newton	89
Louise	93	Russell	83
June	64	Myron	83
Allister	43	Herman	83
Robert S	83	Nadine	78
Thomas	64	John	94
Regina	50	Curtis	94
Kenneth	17	Margaret	61
Carol P	93	Marshall	61
David	87	Frederick	72
Mean	74.88		87.33
Median	82.5		91.5
Q1	93.5		97
Q3	62		80.5
S.D.	21.56		11.80
Number of Pupils	24		24

Difference in means	12.45
Difference in medians	9
Difference in Q1	3.5
Difference in Q3	18.5
Difference in S.D.	9.76



Mean	74.88
Median	82.5
Q1	93.5
Q3	62
% perfect	0
S.D.	21.56
No. of pupils	24

Figure 3a. Graphic representation of Table VI, showing on the May test, the distribution of scores in the manipulative group, together with summary data.



Mean	87.33
Median	91.5
Q1	97
Q3	80.5
% perfect	25
S.D.	11.80
No. of pupils	24

Figure 3b. Graphic representation of Table VI, showing on the May test, the distribution of scores in the objective group, together with summary data.

two had scores between 85 and 89, two between 80 and 84, one between 75 and 79, one between 65 and 69, four between 60 and 64, two between 50 and 54, two between 40 and 44, while one had a score between 15 and 19. Figure 3b shows the distribution of scores in the May test in the objective group; six pupils had scores of 100, six pupils had scores between 90 and 94, two pupils had scores between 85 and 89, four between 80 and 84, two between 75 and 79, two between 70 and 74, and two between 60 and 64.

When comparing figures 3a and 3b, the median, 91.5, of the objective group was 9 points higher than the median, 82.5, of the manipulative group, while the objective group shows a 12.45 advantage in the mean score. The first quartile (Q1) of the objective group is 97, 3.5 points higher than the first quartile, 93.5, in the manipulative group, while the third quartile (Q3) of the objective group, 80.5, is 18.5 points higher than the third quartile, 62, in the manipulative group. The difference in the standard deviations of the two groups is 9.76.

Table VII lists some of the errors common to both groups in the May test.

There were more mistakes in the multiplication and division examples than in the addition and subtraction. The same errors seem apparent in this test as in the previous ones:

The failure to reduce the answer to the lowest terms

Incorrect multiplication, addition, subtraction or division

Table VII listing the errors found in the two groups, manipulative and objective, in the May test.

Addition

1. Using incorrect common denominator, as ?/5 and ?/2 using ?/5
2. Failure to add whole numbers
3. Copied example incorrectly

Subtraction

1. Failure to reduce answer
2. When subtrahend fraction greater than minuend, forgot one had been borrowed from whole number.
3. Copied incorrectly

Multiplication

1. Example left out entirely
2. Lack of accurate knowledge of division facts
   (when canceling)
   (when reducing)
3. Lack of understanding of entire process
4. Lack of accurate knowledge of multiplication facts
5. Adding numerators instead of multiplying
6. Copied example incorrectly
7. Changed improper fraction to mixed number incorrectly in answer, as 5/4 equals 5 1/4
8. Did wrong process, added instead of multiplying

Division

1. Inverted dividend instead of divisor

Table VII (Continued)

2. Reduced answer incorrectly, as 2/8 equals 4, 2/2 equals 2
3. Did not reduce answer
4. Failed to invert divisor, proceeded as though multiplying
5. Inaccurate multiplication
6. Inverted both fractions
7. Forgot the denominator, as 1/2 times 1/2 equals 1

Failure to copy a figure correctly, either when rewriting the parts of the example or the answer

Doing the wrong process.

The same types of mistakes persisted throughout the three tests given in Grade 5. Those occurring most frequently were:

Failure to reduce answers to lowest terms

Copying figures incorrectly

Mistakes in the four fundamental processes

A complete lack of understanding of what was to be done.

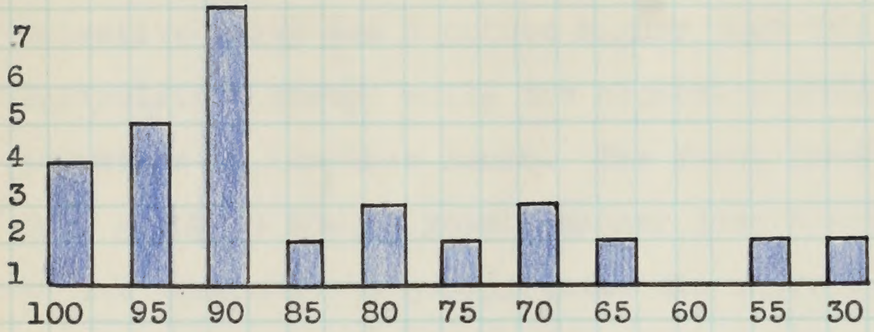
Table VIII shows the scores in each group, together with the summary data, for the test using less useful fractions given in Grade 6. The median of the manipulative group was 90, while the mean of this group was 83.48. The median of the objective group was 95, while the mean was 92.17. The standard deviation of the manipulative group was 16.31, as compared with 8.82 in the objective group. The scores in the manipulative group ranged from 100 to 30, a variation of 70 points, while the scores in the objective group ranged from 100 to 70, a difference of 30 points. In the manipulative group there were three 100's (11.11%), and in the objective group eight 100's (34.78%).

Figure 4a shows the distribution of scores in the test using less useful fractions, given in Grade 6 to the manipulative group. Three pupils had a score of 100, four had scores of 95, seven had 90, one had 85, two had 80, one had 75, two had 70, one had 65, one had 55, and one had 30. Figure 4b shows

Table VIII showing the scores of the two groups, manipulative and objective, in the test using the less useful fractions, given in Grade 6, together with summary data.

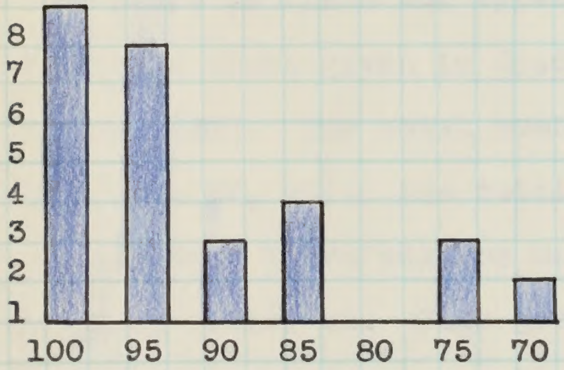
Manipulative		Objective	
Marie	90	James	100
Richard M	90	Carolyn	100
Carol C	95	Robert	85
Ingrid	95	Kathryn	95
James	100	Donald	95
Robert B	75	Craig	95
Betsy	90	Virginia	100
Betty	70	Mary	90
Barbara	95	Gilbert	95
Marjorie	90	Thelma	85
Verne	90	Dorothy	100
Eleanor	80	Henry	100
Phyllis	85	Phyllis	95
Florence	90	Newton	85
Louise	80	Russell	75
June	100	Myron	100
Allister	65	Herman	95
Robert S	55	Nadine	70
Thomas	30	John	100
Regina	95	Curtis	100
Kenneth	90	Margaret	75
Carol P	70	Marshall	90
David	100	Frederick	95
Mean	83.48		92.17
Median	90		95
Q1	95		100
Q3	77.5		87.5
S.D.	16.31		8.82
Number of Pupils	23		23

Difference in means	8.69
Difference in medians	5
Difference in Q1	5
Difference in Q3	10
Difference in S.D.	7.49



Mean	83.48
Median	90
Q1	95
Q3	77.5
% perfect	11.11
S.D.	16.31
No. of pupils	23

Figure 4a. Graphic representation of Table VIII, showing on the test using the less useful fractions given in grade 6, the distribution of scores in the manipulative group, together with summary data.



Mean	92.17
Median	95
Q1	100
Q3	87.5
% perfect	34.78
S.D.	8.82
No. of pupils	23

Figure 4b. Graphic representation of Table VIII, showing on the test using the less useful fractions given in grade 6, the distribution of scores in the objective group, together with summary data.

the distribution of scores in the test using less useful fractions, given in Grade 6, to the objective group. Eight pupils had a score of 100, seven had 95, two had 90, three had 85, two had 75, and one had 70.

When comparing figures 4a and 4b, the median, 95, of the objective group was 5 points higher than the median, 90, of the manipulative group, while the objective group shows a 8.69 advantage in the mean score. The first quartile of the objective group is 100, 5 points higher than the first quartile, 95, in the manipulative group, while the third quartile of the objective group, 87.5, is 10 points higher than Q3, 77.5, in the manipulative group. The difference in the standard deviation of the two groups is 7.49.

Table IX lists the types of errors found in the less useful fractions tests given in Grade 6, classified according to the process used. The total number of errors in the manipulative group was 77, while the total for the objective group was 37. In each case there was one more mistake than the number of examples failed on. Both groups had more mistakes in the multiplication examples. The manipulative group had almost as many mistakes in the other three processes--addition, subtraction, and division. The objective group had almost an equal number of mistakes in the subtraction examples, but had fewer mistakes in the division and addition examples.

Table X shows the scores in each group, together with summary data, for the test using more useful fractions given in

Table IX listing the errors found in the two groups, manipulative and objective, in the test using less useful fractions, given to Grade 6.

	Manipulative	Objective
Addition		
1. Forgot to write part of answer	2	1
2. Used incorrect numerator	4	1
3. Added incorrectly	1	0
4. Forgot to change answer	5	1
5. Changed answer incorrectly	4	1
6. Wrote incorrect common denominator	1	1
7. Failed to understand how to do example	1	0
	18	5
Subtraction		
1. Failed to reduce answer	3	6
2. Did not change to common denominator	0	1
3. Multiplied	0	1
4. When changing to common denominator found incorrect numerator	0	3
5. Subtracted incorrectly	2	0
6. Tried to subtract larger fraction from smaller	5	0
7. Added	3	0
8. Forgot part of denominator	1	0

Table IX (Continued)

		Manipulative	Objective
9.	Reduced answer incorrectly	3	0
		17	11
	N.B. (One example had two mistakes)		
	Multiplication		
1.	Subtracted	3	0
2.	Divided	3	2
3.	Cancelled both denominators	1	0
4.	Multiplied incorrectly	2	1
5.	Failed to reduce answer	3	2
6.	Incorrect division	2	2
7.	Inverted both fractions	1	0
8.	Found only one part when more asked for	2	0
9.	Changed answer incorrectly	0	5
10.	When cancelled still retained denominator	3	0
		20	12
	N.B. (One example had two mistakes)		
	Division		
1.	Changed answer incorrectly	1	3
2.	Failed to invert	7	4
3.	Multiplied incorrectly	1	0
4.	Failed to change answer	2	0
5.	Inverted both fractions	3	0

Table IX (Continued)

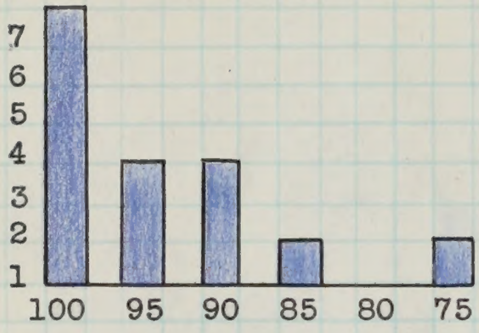
	Manipulative	Objective
6. Canceled (divided) incorrectly	1	1
7. Canceled only numerator	0	1
	15	9
Changing to New Denominator		
1. Did not do example	3	0
2. Incorrect division	1	0
3. Found only one part when more asked for	1	0
4. Changed to different denominator than one asked for	1	0
5. Cannot discover the mistake 1/2 equals 17/8	1	0
	7	0
Total	77	36

Table X showing the scores of the two groups, manipulative and objective, in the test using the more useful fractions, given in Grade 6, together with summary data.

Manipulative		Objective	
Richard M	95	Carolyn	90
Carol C	100	Robert	100
James	95	Donald	95
Betsy	90	Virginia	95
Barbara	100	Gilbert	100
Marjorie	100	Thelma	85
Verne	100	Dorothy	100
Eleanor	100	Henry	100
Florence	100	Newton	90
June	90	Myron	85
Allister	100	Herman	85
Thomas	85	John	95
Regina	95	Curtis	70
Carol P	75	Marshall	90
David	90	Frederick	75

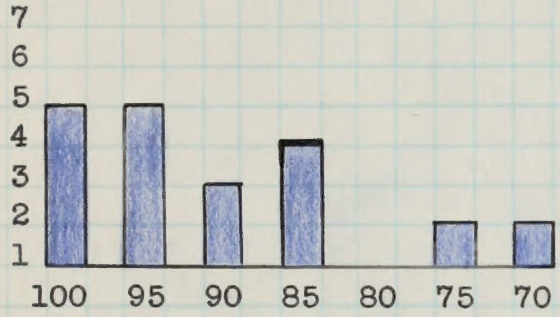
	Manipulative	Objective
Mean	94.33	90.67
Median	95	95
Q1	100	100
Q3	90	85
S.D.	7.04	8.92
Number of Pupils	15	15

Difference in means	3.66
Difference in medians	0
Difference in Q1	0
Difference in Q3	5
Difference in S.D.	1.88



Mean	94.33
Median	95
Q1	100
Q3	90
% perfect	46.67
S.D.	7.04
Number of pupils	15

Figure 5a. Graphic representation of Table X, showing on the test using more useful fractions given in grade 6, the distribution of scores in the manipulative group, together with summary data.



Mean	90.67
Median	95
Q1	100
Q3	85
% perfect	26.67
S.D.	8.92
Number of pupils	15

Figure 5b. Graphic representation of Table X, showing on the test using more useful fractions given in grade 6, the distribution of scores in the objective group, together with summary data.

Grade 6. The median for the manipulative group was 95, while the mean of this group was 94.33. The median of the objective group was 95 and the mean 90.66. The standard deviation of the manipulative group was 7.04, as compared with 8.92 in the objective group. The scores in the manipulative group ranged from 100 to 75, a variation of 25 points, while the scores in the objective group ranged from 100 to 70, a difference of 30 points. In the manipulative group there were seven 100's (46.67%), and in the objective group four 100's (26.67%).

Figure 5a shows the distribution of scores in the test using the more useful fractions when given in Grade 6 to the manipulative group. Seven pupils had scores of 100, three pupils had scores of 95, three had 90, one had 85, and one had 75. Figure 5b shows the distribution of scores on the test using the more useful fractions when given in Grade 6 to the objective group. Four pupils had a score of 100, four had 95, two had 90, three had 85, one had 75, and one had 70.

When comparing figures 5a and 5b, the median, 95, of the objective group was just the same as the median, 95, of the manipulative group, while the manipulative group shows 3.66 advantage in the mean score. The first quartile of the objective group is 100, the same as the first quartile in the manipulative group, while the third quartile of the objective group, 85, is 5 points lower than the third quartile, 90, in the manipulative group. The difference in the standard deviation of the two groups is 1.88.

Table XI lists the types of errors found in the more useful fractions test given in Grade 6, classified according to the process used. The total number of errors in the manipulative group was 17, while the total for the objective group was 28. In the objective group there were more mistakes in the multiplication examples, while in the manipulative group there were more mistakes in the subtraction examples. The only explanation for the great number of mistakes in the objective group in this test seems to be confusion on the part of the sixth grade teacher. The work in this test more closely resembled the work done in the previous year in the fifth grade, and yet they could do the test using less useful fractions and make fewer mistakes. In discussing this with the sixth grade teacher, she said that the class as a whole had a better understanding of fractions, what they mean and their values, than the group the year before, but that they did not seem to know how to handle them as well, and yet this test was not given until after the one using the less useful fractions.

Table XII shows the scores in each group together with the summary data, for the test using the less useful fractions, given in Grade 7. The median of the manipulative group was 100, while the mean of this group was 97.5. The median of the objective group was 100, while the mean was 97.86. The standard deviation of the manipulative group was 3.66, as compared with 3.11 in the objective group. The scores in the manipulative group ranged from 100 to 90, a variation of 10

Table XI listing the errors found in the two groups, manipulative and objective, in the test using more useful fractions, given to Grade 6.

	Manipulative	Objective
Addition		
1. Added incorrectly	1	0
2. Divided incorrectly	1	0
3. Changed answer to incorrect mixed number	1	1
4. Forgot denominator when adding	1	1
5. Added both numerator and denominator	0	1
	4	3
Subtraction		
1. When borrowed just subtracted from the borrowed one without adding the fraction	1	0
2. When needed to borrow to subtract fractions just subtracted whole numbers and left fractions	1	0
3. Forgot that one was borrowed	1	0
4. Copied fraction incorrectly	1	0
5. Subtracted incorrectly	2	2
6. Added instead of subtracting	0	5
7. Borrowed when did not need to	0	2
	6	9

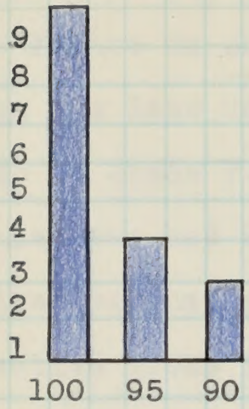
Table XI (Continued)

	Manipulative	Objective
Multiplication		
1. Canceled then added numerators	1	0
2. Added instead of multiplying	2	0
3. Reduced answer incorrectly	1	5
4. Changed answer to incorrect mixed number	0	2
5. Put a denominator under answer	0	2
6. Added denominators	0	1
7. Multiplied incorrect numbers together	0	2
	4	12
Understanding of Fractions		
1. 1/2 equals 4/4	1	1
2. Divided fraction into wrong number of parts	1	0
3. Counted parts of chocolate bar incorrectly	1	1
4. Could not tell larger fraction from smaller	0	2
	3	4
Total	17	28

Table XII showing the scores of the two groups, manipulative and objective, in the test using the less useful fractions, given in Grade 7, together with summary data.

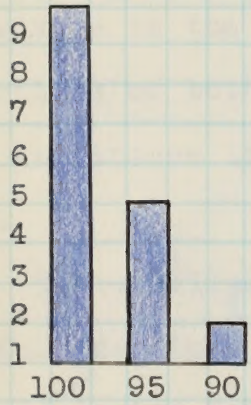
Manipulative		Objective	
Richard M	100	Carolyn	100
Carol C	100	Robert	95
James	100	Donald	100
Betsy	100	Virginia	100
Barbara	100	Gilbert	100
Marjorie	100	Thelma	95
Verne	100	Dorothy	100
Eleanor	100	Henry	95
Florence	90	Newton	100
June	100	Myron	100
Allister	95	Herman	100
Thomas	90	John	100
Regina	95	Curtis	90
Carol P	95	Marshall	95
Mean	97.5		97.86
Median	100		100
Q1	100		100
Q3	95		95
S.D.	3.66		3.11
Number of Pupils	14		14

Difference in means	.36
Difference in medians	0
Difference in Q1	0
Difference in Q3	0
Difference in S.D.	.55



Mean	97.5
Median	100
Q1	100
Q3	95
% perfect	64.29
S.D.	3.66
Number of pupils	14

Figure 6a. Graphic representation of Table XII, showing on the test using the less useful fractions given in grade 7, the distribution of scores in the manipulative group, together with summary data.



Mean	97.86
Median	100
Q1	100
Q3	95
% perfect	64.29
S.D.	3.11
Number of pupils	14

Figure 6b. Graphic representation of Table XII, showing on the test using the less useful fractions given in grade 7, the distribution of scores in the objective group, together with summary data.

points, while the scores in the objective group had the same range, 100 to 90, with a difference of 10 points. In the manipulative group there were nine 100's (64.29%) and in the objective group the same, nine 100's (64.29%).

Figure 6a shows the distribution of scores in the test using the less useful fractions, given to the manipulative group in Grade 7. Nine pupils had a score of 100, three had 95, and two had 90. Figure 6b shows the distribution of scores in the test using the less useful fractions, given to the objective group in Grade 7. Nine pupils had a score of 100, four had 95, and one had 90.

When comparing figures 6a and 6b, the median, 100, of the objective group was just the same as the median, 100, of the manipulative group, while the objective group shows a .36 advantage in the mean score. The first quartiles and third quartiles of both groups are the same, 100 and 95 respectively. The difference in the standard deviation of the two groups is .55.

Table XIII lists the types of errors found in the less useful fractions test given in Grade 7. The total number of errors in the manipulative group was 8, while the total for the objective group was 6. In the manipulative group there was one more mistake than the number of examples failed on, i.e., one example had two mistakes. There were no mistakes in the division of fractions examples.

Table XIV shows the scores in each group, together with

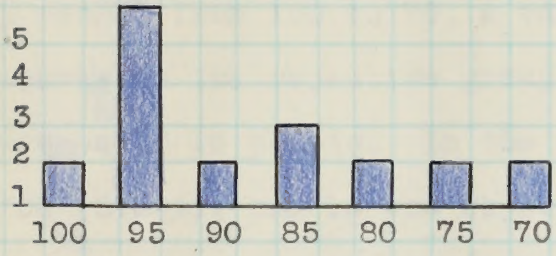
Table XIII listing the errors found in the two groups, manipulative and objective, in the test using less useful fractions, given to Grade 7.

	Manipulative	Objective
Addition		
1. Changed answer to incorrect mixed number	1	0
	1	0
Subtraction		
1. Added instead of subtracting	2	0
2. Failed to reduce answer	1	1
3. Reduced answer incorrectly	0	1
4. When changing to common denominator found incorrect numerator	2	0
	5	2
N.B. (One example had two mistakes)		
Multiplication		
1. Multiplied incorrectly	0	2
2. Reduced answer incorrectly	1	1
3. Canceled (divided) incorrectly	0	1
4. Forgot the denominator was canceled	1	0
	2	4
Total	8	6

Table XIV showing the scores of the two groups, manipulative and objective, in the test using the more useful fractions, given in Grade 7, together with summary data.

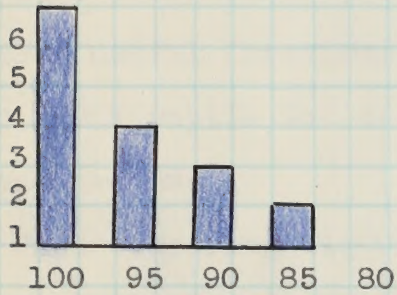
Manipulative		Objective	
Richard M	85	Carolyn	100
Carol C	90	Robert	95
James	85	Donald	100
Barbara	95	Gilbert	95
Marjorie	95	Thelma	90
Verne	95	Dorothy	100
Florence	80	Newton	100
June	95	Myron	100
Allister	95	Herman	85
Thomas	75	John	100
Regina	100	Curtis	90
Carol P	70	Marshall	95
Mean	88.33		95.83
Median	92.5		97.5
Q1	95		100
Q3	82.5		92.5
S.D.	8.98		4.93
Number of Pupils	12		12

Difference in means	7.50
Difference in medians	5
Difference in Q1	5
Difference in Q3	10
Difference in S.D.	4.05



Mean	88.33
Median	92.5
Q1	95
Q3	82.5
% perfect	8.33
S.D.	8.98
Number of pupils	12

Figure 7a. Graphic representation of Table XIV, showing on the test using the more useful fractions given in grade 7, the distribution of scores in the manipulative group, together with summary data.



Mean	95.83
Median	97.5
Q1	100
Q3	92.5
% perfect	50
S.D.	4.93
Number of pupils	12

Figure 7b. Graphic representation of Table XIV, showing on the test using the more useful fractions given in grade 7, the distribution of scores in the objective group, together with summary data.

summary data, for the test using more useful fractions, given in Grade 7. The median for the manipulative group was 92.5, while the mean of this group was 88.33. The median of the objective group was 97.5, and the mean 95.83. The standard deviation of the manipulative group was 8.98, as compared with 4.93 in the objective group. The scores in the manipulative group ranged from 100 to 70, a variation of 30 points, while the scores in the objective group ranged from 100 to 85, a difference of 15 points. In the manipulative group there was one 100 (8.33%), and in the objective group six 100's (50%).

Figure 7a shows the distribution of scores on the test using the more useful fractions when given in Grade 7 to the manipulative group. One pupil had a score of 100, five had 95, one had 90, two had 85, one had 80, one had 75, and one had 70. Figure 7b shows the distribution of scores on the test using the more useful fractions when given in Grade 7 to the objective group. Six pupils had a score of 100, three had 95, two had 90, and one had 85.

When comparing figures 7a and 7b the median, 97.5, of the objective group was 5 points higher than the median, 92.5, of the manipulative group, while the objective group shows a 7.50 advantage in the mean score. The first quartile of the objective group is 100, five points higher than the first quartile, 95, in the manipulative group, while the third quartile of the objective group, 92.5, is 10 points higher than the third quartile, 82.5, in the manipulative group. The

difference in standard deviation of the two groups is 4.05.

Table XIV-A shows the chronological age, mental age, and intelligence quotient of the pairs of children who completed Grade 7. The manipulative group ranges from an I.Q. of 118 to 88 with a median I.Q. of 102 and a mean I.Q. of 103.08, while the objective group ranges in I.Q. from 119 to 82, median I.Q. 104 and mean I.Q. 102.92. The median C.A. of the manipulative group is 10 years 7-1/2 months, while the median C.A. of the objective group is 10 years 5-1/2 months. The mean C.A. of the manipulative group is 10 years 7 months, and of the objective group 10 years 9 months. The median M.A. of the manipulative group is 10 years 7-1/2 months, the mean M.A. 10 years 11 months, while the median M.A. of the objective group is 11 years and the mean M.A. 11 years 1 month.

Table XV lists the types of errors found in the more useful fractions tests, given in Grade 7, classified according to the process used. The total number of errors in the manipulative group was 28, while the total for the objective group was 10. In the objective group, there were the same number of errors in the subtraction and multiplication examples. In the manipulative group, there were 13 mistakes in the subtraction examples and 9 in the multiplication examples.

Table XVI shows the complete record of scores for the manipulative group.

Table XVII shows the complete record of scores for the objective group.

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Table XIV-A showing the pairing of the groups, manipulative and objective, left at the end of Grade 7, with the chronological age (C.A.), mental age (M.A.), and intelligence quotient (I.Q.), together with summary data.

Manipulative				Objective			
	C.A.	M.A.	I.Q.		C.A.	M.A.	I.Q.
Richard M	9-10	11-10	118	Carolyn	10- 3	12- 2	119
Carol C	10- 4	12- 2	116	Robert	10- 5	12- 4	118
James	10- 9	12- 5	114	Donald	10- 6	12	114
Barbara	10- 4	11- 3	108	Gilbert	10- 3	11	107
Marjorie	10- 2	10-10	106	Thelma	10- 3	11	107
Verne	9-10	10- 4	105	Dorothy	10-10	11- 6	106
Florence	10- 7	10- 5	99	Newton	10- 3	10- 5	102
June	10- 8	10- 4	97	Myron	11- 5	11- 5	100
Allister	11- 3	10-11	97	Herman	11	10-11	99
Thomas	10-11	10- 4	95	John	11	10	91
Regina	10- 8	10	94	Curtis	11- 2	10	90
Carol P	11- 6	10	88	Marshall	12- 3	10	82

	C.A.	M.A.	I.Q.	C.A.	M.A.	I.Q.
Median	10-7½	10- 7½	102	10-5½	11	104
Mean	10-7	10-11	103.08	10-9	11- 1	102.92

	Manipulative	Objective
Number of boys	4	9
Number of girls	8	3
Total	12	12

Table XV listing the errors found in the two groups, manipulative and objective, in the test using more useful fractions, given to Grade 7.

	Manipu-lative	Objec-tive
Addition		
1. Copied answer incorrectly	1	0
2. Did not carry	1	0
3. Called the denominator of fraction the whole number for answer	1	0
	3	0
Subtraction		
1. When borrowed subtracted from the one borrowed, did not add the fraction	3	1
2. Added instead of subtracting	3	1
3. Copied incorrect answer	1	0
4. Forgot one had been borrowed	1	1
5. Subtracted incorrectly	5	1
6. Failed to reduce answer	0	1
	13	5
Multiplication		
1. When canceled, forgot to cancel denominator	2	0
2. Added denominator	2	0
3. Added numerator and denominator	1	0
4. Multiplied incorrectly	2	2

Table XV (Continued)

	Manipulative	Objective
5. Divided incorrectly	2	0
6. Added incorrectly	0	1
7. Divided denominators	0	1
8. Inverted two figures in answer	0	1
	9	5
Understanding of Fractions		
1. Colored 2/6 instead of 5/6	1	0
2. Can cut 1/2 into 4/4	1	0
3. Counted parts of chocolate bar incorrectly	1	0
	3	0
Total	28	10

Table XVI showing complete list of scores for manipulative group.

	Grade 5			Grade 6		Grade 7	
	Oct.	Feb.	May	Use-less	Use-ful	Use-less	Use-ful
Marie	100	100	97	90	100	100	95
Richard M	80	100	94	90	95	100	85
Carol C	100	100	93	95	100	100	90
Ingrid	90	80	94	95			
James	100	80	98	100	90	100	85
Robert B	100	80	43	75		100	80
Betsy	100	90	52	90	95	100	
Betty	100	80	60	70			
Barbara	90	100	82	95	100	100	95
Marjorie	100	90	98	90	100	100	95
Verne	100	70	64	90	100	100	95
Richard H	100	100	94	95	100	100	100
Eleanor	80	90	68	80	100	100	
Phyllis	99	80	88	85			
Florence	100	90	78	90	100	90	80
Louise	80	90	93	80			
June	60	60	64	100	90	100	95
Allister	90	70	43	65	100	95	95
Robert S	100	80	83	55			
Thomas	80	80	64	30	85	90	75
Regina	90	70	50	95	95	95	100
Kenneth	79	90	17	90			
Carol P	100	70	93	70	75	95	70
David	100	80	87	100	90	100	80

Table XVII showing complete list of scores for objective group.

	Grade 5			Grade 6		Grade 7	
	Oct.	Feb.	May	Useless	Useful	Useless	Useful
James	100	100	100	100			
Carolyn	96	80	100	100	90	100	100
Robert	90	60	78	85	100	95	95
Kathryn	100	90	100	95	100	100	100
Donald	90	90	94	95	95	100	100
Craig	60	90	100	95	75		
Virginia	88	80	100	100	95	100	95
Mary	88	50	94	90	70	100	
Gilbert	100	100	94	95	100	100	95
Thelma	100	90	89	85	85	95	90
Dorothy	100	100	100	100	100	100	100
Barbara	100	90	94				
Henry	98	100	83	100	100	95	100
Phyllis	100	100	72	95	75	95	85
Newton	100	100	89	85	90	100	100
Russell	80	70	83	75	90	95	100
Myron	86	90	83	100	85	100	100
Herman	100	90	83	95	85	100	85
Nadine	100	70	78	70	95	100	95
John	100	100	94	100	95	100	100
Curtis	100	90	94	100	70	90	90
Margaret	98	40	61	75	30		
Marshall	100	100	61	90	95	95	95
Frederick	100	70	72	95	75		

Table XVIII gives the summary data from Tables II, IV, VI, VIII, X, XII, and XIV.

In 1935, an experiment somewhat similar to this study was carried on in the sixth grades in another city, called city B, on the opposite side of Boston. There were many more children used in this experiment. Its purpose was "to attempt to determine whether or not children benefit more when all the time for fractions is spent on useful fractions or when part of the time is devoted to fractions beyond common usage."

The experiment was to last thirty-two half hour periods. The experimental group was taught only the useful fractions, while the control group was taught both useful fractions and those beyond adult usage.

Converting the figures to compare with those in this study, Table XIX shows the scores and frequency, in the control group, for a test using useful fractions, given in city B, together with summary data. The median for the control group was 80, while the mean of this group was 73.8. The scores ranged from 100 to 0. There were sixty-three 100's or 11.37% perfect scores.

Table XX shows the scores and frequency in the experimental group for a test using useful fractions, given in city B, together with summary data. The median of this group was 80, while the mean was 77.3. The scores ranged from 100 to 10. There were sixty-four 100's or 10.72% perfect scores.

Figure 8a shows the distribution of scores in the control

Table XVIII showing the summary data.

Grade	Fig.	Median	Mean	Q1	Q3	% Perfect	S.D.	No. Pupils
5	1a	100	92.42	100	85	54.17	10.52	24
	1b	100	94.75	100	90	58.33	9.23	24
	2a	80	84.17	90	80	20.83	11.15	24
	2b	90	85	100	75	33.33	16.43	24
	3a	82.5	74.88	93.5	62	0	21.56	24
	3b	91.5	87.33	97	80.5	25	11.80	24
6	4a	90	83.48	95	77.5	11.11	16.31	23
	4b	95	92.17	100	87.5	34.78	8.82	23
	5a	95	94.33	100	90	46.67	7.04	15
	5b	90	90.67	100	85	26.67	8.92	15
7	6a	100	97.5	100	95	64.29	3.66	14
	6b	100	97.86	100	95	64.29	3.11	14
	7a	92.5	88.33	95	82.5	8.33	8.98	12
	7b	97.5	95.83	100	92.5	50	4.93	12

Table XIX showing the scores in the control group together with summary data, for the test using useful fractions and given in city B.

Control Group

Score	Frequency
100	63
95	54
90	60
85	61
80	58
75	47
70	28
65	51
60	28
55	31
50	16
45	16
40	11
35	5
30	5
25	8
20	4
15	4
10	2
5	1
0	1

Mean	73.8
Median	80
Q1	85
Q3	60
% Perfect	11.37
Number of Pupils	554

Table XX showing the scores in the experimental group together with summary data, for the test using useful fractions and given in city B.

Experimental Group

Score	Frequency
100	64
95	66
90	85
85	74
80	55
75	45
70	49
65	38
60	34
55	21
50	20
45	16
40	11
35	5
30	4
25	6
20	1
15	2
10	1

Mean	77.3
Median	80
Q1	90
Q3	65
% Perfect	10.72
Number of Pupils	597

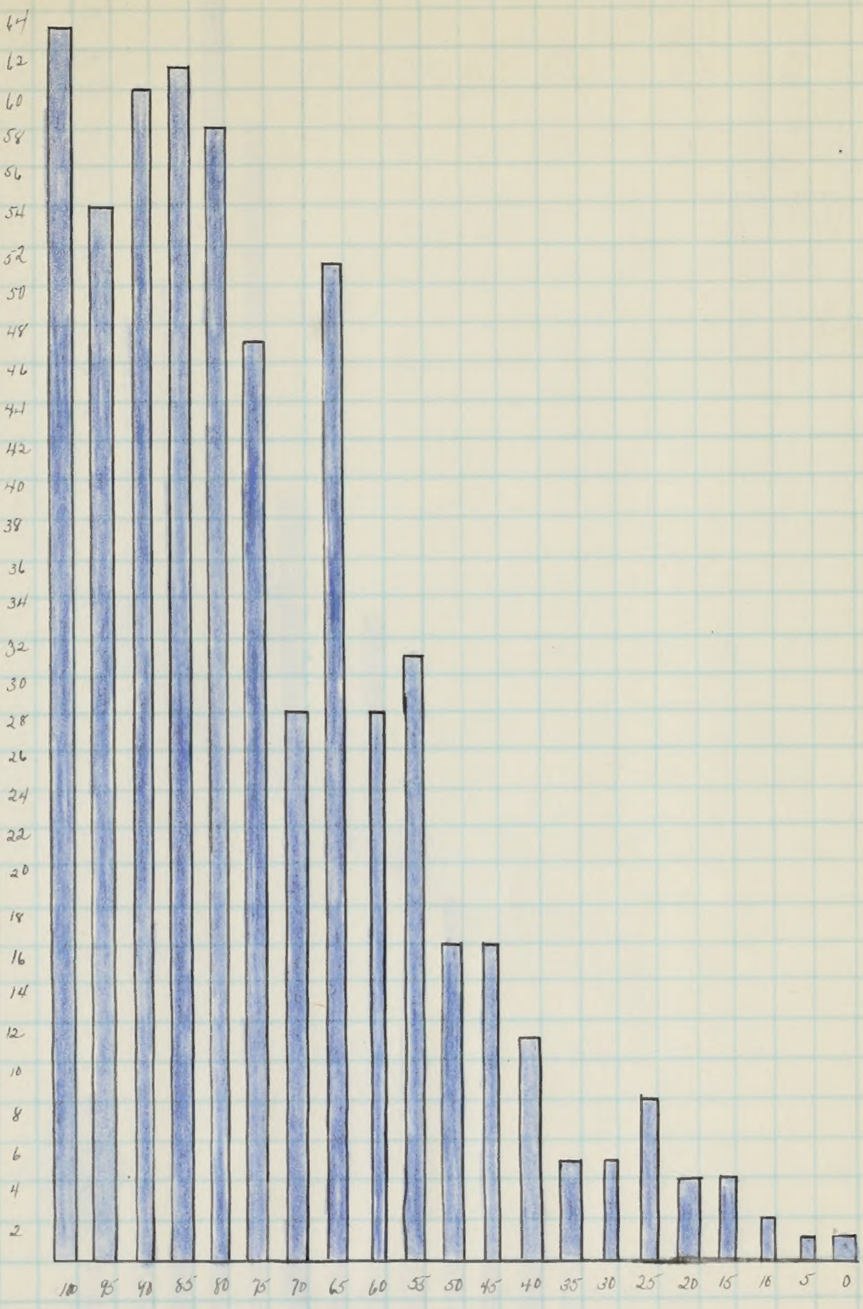

Figure 8a. Graphic representation of Table XIX,showing the disribution of scores in a useful fraction test, given to the control group in city B.

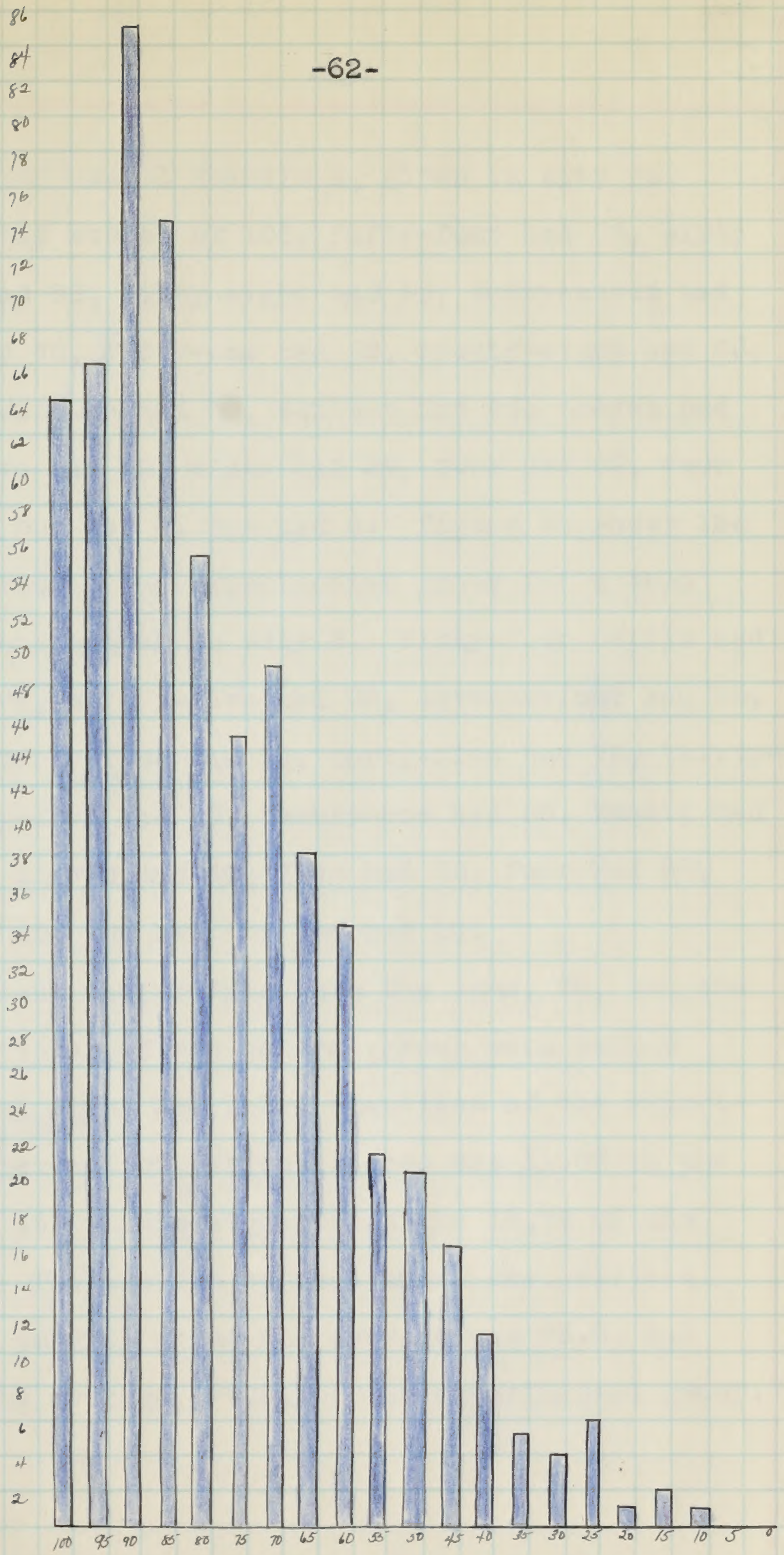

Figure 8b. Graphic representation of Table XX, showing the distribution of scores in a useful fraction test, given to the experimental group in city B.

group for a test using useful fractions, given in city B. Sixty-three pupils had scores of 100, fifty-four had 95, sixty had 90, sixty-one had 85, fifty-eight had 80, forty-seven had 75, twenty-eight had 70, fifty-one had 65, twenty-eight had 60, thirty-one had 55, sixteen had 50, sixteen had 45, eleven had 40, five had 35, five had 30, eight had 25, four had 20, four had 15, two had 10, one had 5, one had 0. Figure 8b shows the distribution of scores in the experimental group for a test using useful fractions, given in city B. Sixty-four pupils had 100, sixty-six had 95, eighty-five had 90, seventy-four had 85, fifty-five had 80, forty-five had 75, forty-nine had 70, thirty-eight had 65, thirty-four had 60, twenty-one had 55, twenty had 50, sixteen had 45, eleven had 40, five had 35, four had 30, six had 25, one had 20, two had 15, one had 10.

In this experiment the medians were the same, 80, but the first and third quartiles of the control group were each 5 points lower than the first and third quartiles of the experimental group. The percent of perfect scores was 11.37 in the control group, just .65 points higher than the 10.72 of the experimental group. The average or mean shows the greatest difference. In the control group the average was 73.8, 3.5 points lower than the average, 77.3, of the experimental group.

CHAPTER IV. COMPARISONS AND SUMMARY

Table XXI shows the total number of mistakes made in the addition, subtraction, and multiplication examples only on the four tests given in Grades 6 and 7, after the work of actually teaching the fraction processes had been introduced and carried through Grade 5 as explained in Chapter II.

The manipulative group failed on 100 examples; of these 26 were addition, 39 were subtraction, and 35 were multiplication. The children with lower I.Q.'s tended to make more mistakes than the children with higher I.Q.'s. The objective group failed on 67 examples; of these 8 were addition, 27 were subtraction, and 32 were multiplication. 100 or 7.81% of a possible 1,280 examples were failed on in the manipulative group, while 67 or 5.23% of a possible 1,280 examples were failed on in the objective group. The manipulative group had more failures in subtraction examples, while the objective group had more failures in multiplication examples.

Table XXII lists all the mistakes made in addition, subtraction, and multiplication in the four tests given in Grades 6 and 7. It is a summary of Tables IX, XI, XIII, and XV. The four columns tell the number of that particular error in the figure from which it was taken. In this list of errors, the failure to reduce to lowest terms was considered as one error, while failure to change an improper fraction to a mixed number was listed as another separate error. As might be expected, the manipulative group had more errors in addition and subtraction

Table XXI showing the number of errors, in each test given in Grades 6 and Grades 7, in addition, subtraction, and multiplication, and the differences.

	Grade 6		Grade 7		
	Less useful test	More useful test	Less useful test	More useful test	Totals
Addition					
Manipulative	18	4	1	3	26
Objective	5	3	0	0	8
Difference	13	1	1	3	18
Subtraction					
Manipulative	16	6	4	13	39
Objective	11	9	2	5	27
Difference	5	3	2	8	18
Multiplication					
Manipulative	20	4	2	9	35
Objective	11	12	4	5	32
Difference	9	8	2	4	3

Total number of mistakes - manipulative 100

Total number of mistakes - objective 67

Difference 33

Table XXII, a summary of Tables IX, XI, XIII, and XV.

List of All Errors

	Tables IX	XI	XIII	XV	Manipulative	Objective	Total
Addition							
1. Forgot to write part of answer	1	4		3	3	2	5
2. Used incorrect numerator	2				4	1	5
3. Added incorrectly	3	1			3	0	3
4. Forgot to change answer	4				5	1	6
5. Changed answer incorrectly	5	3	1		6	2	8
6. Wrote incorrect common denominator	6				1	1	2
7. Failed to understand how to do example	7				1	0	1
8. Divided incorrectly		2			1	0	1
9. Added both numerator and denominator		5			0	1	1
10. Copied answer incorrectly				1	1	0	1
11. Did not carry				2	2	3	5
Subtraction							
1. Failed to reduce answer	1		2	6	4	8	12
2. Did not change to common denominator	2				0	1	1
3. Multiplied	3				0	1	1

Table XXII (Continued)

	Tables						
	IX	XI	XIII	XV	Manipulative	Objective	Total
4. When changing to common denominator, found incorrect numerator	4		4		2	3	5
5. Subtracted incorrectly	5	5		5	8	3	11
6. Tried to subtract larger fraction from smaller	6				7	0	7
7. Added	7	6	1	2	5	6	11
8. Forgot part of denominator	8				1	0	1
9. Reduced answer incorrectly	9		3		3	1	4
10. When borrowed just subtracted from borrowed one without adding fraction		2			4	1	5
11. When needed to borrow to subtract fractions, just subtracted whole numbers, not fractions		2			1	0	1
12. Forgot one was borrowed		3		4	2	1	3
13. Copied fraction incorrectly		4		3	2	0	2
14. Borrowed when did not need to		7			0	2	2
Multiplication							
1. Subtracted	1				3	0	3

Table XXII (Continued)

		Tables						
		IX	XI	XIII	XV	Manipulative	Objective	Total
2.	Divided	2				3	2	5
3.	Cancelled both denominators	3				1	0	1
4.	Multiplied incorrectly	4		1	3	3	3	6
5.	Failed to reduce answer	5				3	2	5
6.	Incorrect division	6		3	5	4	3	7
7.	Inverted both fractions	7				1	0	1
8.	Found only one part when more asked for	8				2	0	2
9.	Changed answer incorrectly	9	4			0	7	7
10.	When cancelled still retained denominator	10				3	0	3
11.	Cancelled, then added numerators		1			1	0	1
12.	Added		2			2	0	2
13.	Reduced answer incorrectly		3	2		2	6	8
14.	Put a denominator under answer		5	4	1	3	2	5
15.	Added denominators		6		2	2	1	3
16.	Multiplied incorrect numbers together		7			0	2	2
17.	Multiplied incorrectly				4	2	2	4
18.	Added incorrectly				6	0	1	1

Table XXII (Continued)

		Tables						
		IX	XI	XIII	XV	Manipulative	Objective	Total
19.	Divided denominators				7	0	1	1
20.	Inverted two figures in answer				8	0	1	1
	Total					101	71	172
	Difference						30	

than the objective group, but about the same number of errors in multiplication.

The three most common errors in addition in the manipulative group were--changed answer incorrectly, forgot to change answer, and used incorrect numerator. This accounted for 15 of the total number of errors. The two most common errors in addition in the objective group were--changed answer incorrectly and forgot to write part of answer. This accounted for four of the total number of errors. The manipulative group had a total of 26 errors in addition, while the objective group had a total of 8.

In subtraction, the three most common errors in the manipulative group were--subtracted incorrectly, tried to subtract the larger fraction from the smaller, and added instead of subtracting. This accounted for 20 of the total number of errors in subtraction. In the objective group the two most common errors were--failure to reduce answer and added instead of subtracting. This accounted for 14 of the total number of errors in subtraction. The manipulative group had 39 errors in subtraction, while the objective group had 27.

In the manipulative group there were no outstanding errors in multiplication. There were 4 mistakes in division facts, and each of the following occurred 3 times--subtracted instead of multiplying, divided instead of multiplying, multiplied incorrectly, failed to reduce answer, when cancelling still retained same denominator, and put a denominator under answer.

This accounted for 22 of the errors in multiplication. The two most common errors in the objective group were--changed answer incorrectly and reduced answer incorrectly. Two other rather common errors were errors in multiplication and division. These accounted for 19 of the total number of errors. The manipulative group had a total of 35 errors while the objective group had a total of 33 errors.

Figure 9 includes the twelve pairs of children taking all four of the tests in Grades 6 and 7. At the left are the numbers as taken from the list of errors in Table XXII. In each square was put the total number of times each child made the listed errors. The manipulative group mistakes and totals are in red and the objective group mistakes and totals are in blue.

Studying these mistakes made by the pairs of children to see what the similarities in mistakes were, it was found that in addition only one pair made the same mistake. Regina and Curtis each changed an answer incorrectly. Regina did this in the October test in Grade 7, while Curtis made this mistake in the December test in Grade 6. All the other mistakes were different. All Regina's mistakes were made in addition examples. Once she made a mistake in a division fact, once she used an incorrect common denominator, but these each occurred in different tests.

Errors from Table XXII	Richard	Carolyn	Carole	Robert	James	Donald	Barbara	Gilbert	Marjorie	Thelma	Verne	Dorothy	Florence	Newton	June	Myron	Allister	Herman	Thomas	John	Regina	Curtis	Carol P.	Marshall	Totals		
																									Manipulative	Objective	
Add 1																			1					1	1	1	2
2	1																		1						2	0	2
3													1		1								1		3	0	3
4																	3								3	0	3
5	1												1		1						1	1	1		5	1	6
6										1											1				1	1	2
7																									0	0	0
8																					1				1	0	1
9																									0	0	0
10					1																				1	0	1
11													1												1	0	1
Totals																									18	3	21
Subtract 1						1	1							1			1	1				1	1		3	4	7
2																									0	0	0
3																									0	0	0
4								1		1							1								1	2	3
5			1	1	2				1				2				1	1	1				1		9	2	11
6																			2						2	0	2
7							1			1			1	2					2			1	1	1	5	5	10
8																									0	0	0
9																						1	1		1	1	2
10	2				1													1	1						4	1	5
11																							1		1	0	1
12										1															0	1	1
13													1		1										2	0	2
14										1								1							0	2	2
Totals																									28	18	46
Multiply 1																			3						3	0	3
2																									0	0	0
3																									0	0	0
4				1									1						1			1		1	2	3	5
5									1		1												1		3	0	3
6	1			1							2			1										1	3	3	6
7																			1						1	0	1
8																	1								1	0	1
9				3						1												2			0	6	6
10																							2		2	0	2
11																			1						1	0	1
12	1																		1						2	0	2
13						1				2						1		1	1			1			1	6	7
14																2			1				2		3	2	5
15			1												1									1	2	1	3
16																		1		1					0	2	2
17										1									2			1			2	2	4
18								1																	0	1	1
19																		1							0	1	1
20																									0	0	0
Totals																									26	27	53
	6	0	2	6	4	2	2	2	2	9	3	0	8	4	4	3	7	7	19	1	3	9	12	5			

Figure 9. Showing the errors of the twelve pair of children taking all four tests in Grades 6 and 7. The manipulative group is in red.

In the subtraction examples, there were a few more similarities. Allister and Herman in the test given in October in Grade 6 each failed to reduce an answer, and in the test given in December in Grade 7, they each made a mistake in subtraction. All further mistakes were different. Allister, in the first test given in October in Grade 6, made one mistake three times. He failed to change his answers to mixed numbers. In this same test in multiplication he found only one part of a number when more than one part was asked for. In the October test in Grade 7, he made a mistake in a numerator when he changed his fractions to common denominators. In the December test in the same grade he made a mistake in subtraction itself. Herman had no mistakes in addition examples. Besides the two which he and Allister both made, he made mistakes in borrowing, as when borrowed subtracted from the borrowed one without using the fraction, and borrowed once when he did not need to. His other three mistakes were mistakes in multiplication examples, reduced answer incorrectly and multiplied incorrect numbers together in the December test in Grade 6, and in the December test in Grade 7, divided denominators.

Carol C. and Robert each made one mistake in subtraction. Carol made hers in the October test in Grade 6 and Robert made his in the December test in Grade 6. Carol had one further mistake that occurred in the December test in Grade 7. This time she added denominators instead of multiplying. Robert had no further mistakes in subtraction, but made several in

multiplication. In the October test in Grade 6, once he multiplied incorrectly, also three answers were changed incorrectly. In the October test in Grade 7 he made a mistake in a division fact.

Two pairs of children added a subtraction example--Florence and Newton, Carol P. and Marshall. Florence made this mistake in the October test in Grade 7. Newton made this same mistake twice in the December test in Grade 6. Florence had three mistakes in addition. In the test in October in Grade 6, she added incorrectly and changed an answer incorrectly. In the December test in Grade 7, she failed to carry a whole number. In subtraction, besides the addition mistake, she also made a mistake in subtraction in both tests given in Grade 7. In the December test in Grade 7 she copied a fraction incorrectly. Newton had fewer mistakes. He made no mistakes in addition. In the October test in Grade 6, in subtraction he failed to reduce an answer and in multiplication he made a mistake on a division fact.

The other pair, who added instead of subtracting, Carol P. and Marshall, made this mistake in the December test given in Grade 7. Carol P. had more mistakes than Marshall. In the October test in Grade 7, in addition, she changed an answer incorrectly; in subtraction, she made a mistake in a subtraction fact and reduced an answer incorrectly; in multiplication, she failed to reduce an answer and retained her denominator after cancelling in two examples. In the December test in Grade 6,

in addition she added incorrectly; in subtraction, after borrowing, she subtracted only the whole numbers and forgot the fractions. In the October test in Grade 7, in the subtraction examples, she failed to reduce an answer. In the December test in Grade 7, besides the mistake in subtraction, she put a denominator under two of the examples. Marshall had a mistake in a division fact in the multiplication examples and forgot to write part of an answer in the October test in Grade 6. In the December test in Grade 6, in one multiplication example, he added the denominators. In the October test in Grade 7, in the multiplication examples, he multiplied incorrectly.

In the same way the other pairs may be studied through and compared as to kinds of errors. Few of them are similar.

Summary

From the previous discussions it may be seen that

1. The children with higher I.Q.'s tended to make fewer mistakes than those with lower I.Q.'s.
2. In both groups fewer mistakes were made when in Grade 7 than in Grades 5 or 6. This may be due to greater maturity or that the children had more use for fractions.
3. In the total number of individual errors the manipulative group made more errors than the objective group.

4. In addition and subtraction, the objective method resulted in fewer mistakes, when the groups were tested, than the manipulative method.
5. In multiplication, there was very little difference in the total number of errors.
6. The errors made by the manipulative group were, in multiplication, quite different from those made by the objective group. In addition the mistakes were similar--changed answers incorrectly and forgot to change answers. In subtraction the most common errors in the manipulative group were mistakes in subtraction facts, while those in the objective group were mistakes in process, adding instead of subtracting.

   In multiplication, the errors in the manipulative group were more scattered. The one occurring most often was the mistake in division facts. The others were--subtrac-

ting instead of multiplying, dividing instead of multiplying, multiplying incorrectly, failing to reduce answer, forgetting to cancel a denominator, putting a denominator under the answer. The two errors occurring most often in the objective group were changing answer incorrectly and reducing answer incorrectly. The failure to reduce or change answers should be of little concern, but the errors in the facts in the four fundamental processes are mistakes that should be the concern of any teacher interested in 100% accuracy.

7. The results show no significant advantage of one group over the other. If well taught the results will probably be the same whatever the method used.
8. The city B results seem to corroborate the data found in this study.
9. Finally, the mental attitude of the children studying fractions objec-

tively was one of such enjoyment that, even though the test results show no great superiority of ability to work with fractions, this alone would justify using the objective method.

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APPENDIX

APPENDIX

Tests given to each group in the fall. (Results shown graphically in figures 1a and 1b.)

Grade 5

October (Manipulative Group)

1. 6/7= /14
   1/3 = /6
   2/3 = /9
   3/4 = /8
   5/6 = /18
2. 5/6 of 24
3. 2/3 of 213
4. 7/8 of 32
5. 2/4 of 96
6. 3/4 of 24
7. 2/5 of 25
8. 3/8 of 96
9. 3/6 of 18
10. 5/8 of 40

October (Objective Group)

1. 2/3 of 15
2. 1/8 of 32
3. 1/2 of 10
4. 5/8 of 16
5. 3/4 of 16
6. 1/4 of 36
7. 1/2 of 18
8. 5/6 of 24
9. 3/4= /12, 2/3 = /6, 1/3 = /12
   3/4= /8, 5/8= /16, 3/4= /8
   1/3= /6, 5/6= /12

Test given to both groups in February. (Results shown graphically in figures 2a and 2b.)

Grade 5

February (Addition and Subtraction)

1. 9 1/2 + 6 2/3 + 8 5/6
2. 2 3/4 + 6 2/3 + 7 1/2
3. 8 3/4 + 7 1/2 + 6 5/8
4. 4 1/2 + 7 2/5 + 2 3/10
5. 2 3/4 + 1 7/8 + 3 1/2

6. 9 2/3 - 6 1/2
7. 8 1/2 - 6 3/4
8. 10 3/4 - 6 3/8
9. 6 2/3 - 4 2/6
10. 5 - 4 2/3

Final test in Grade 5. (Results shown graphically in figures 3a and 3b.)

Grade 5

May ("A Fraction Review")

1. 1/3= /6 17/5=

 8/12= /3 1 3/5=

2. 3 5/8 + 2 3/4 + 1 1/2
3. 3 4/5 + 2 1/2 + 5
4. 4 5/8 - 3 1/2
5. 5 1/3 - 2 2/3
6. 6 1/3 - 1 5/6
7. 3 1/3 x 2/5
8. 4 1/6 x 1 1/5
9. 2 1/2 x 1/2
10. 5 ÷ 1/2
11. 5 ÷ 1/5
12. 5 ÷ 1/25
13. 1/2 ÷ 1/4
14. 1/2 ÷ 1/8
15. 1/2 ÷ 1/2
16. 1/4 ÷ 1/8
17. 1/8 ÷ 1/2
18. 1/2 ÷ 2

Test given in the fall to each group when in Grade 6 and again in Grade 7. (Results shown graphically in figures 4a and 4b, 6a and 6b.)

Grades 6 and 7

October

Add:

1. 1/2 + 3/8
2. 5/12 + 2/3
3. 1/2 + 4/5
4. 1/6 + 1/2

Subtract:

5. 1/2 - 3/8
6. 2/3 - 5/12
7. 7/10 - 2/5
8. 15/16 - 3/4

Multiply:

9. 5/6 x 1/2
10. 1/2 x 3/8
11. 2/3 x 5/12
12. 4/5 x 15/16

Divide:

13. 3/4 ÷ 1/2
14. 1/2 ÷ 1/4
15. 7/8 ÷ 1/2

Find:

16. 1/9 of 72
17. 3/5 of 75

Fill in the blank numerator:

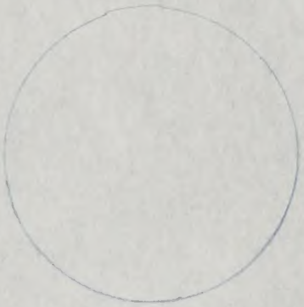
18. 1/4= /16   19. 2/3= /12   20. 1/2= /8

Test given in December to each group when in Grade 6 and again in Grade 7. (Results shown graphically in figures 5a and 5b, 7a and 7b.)

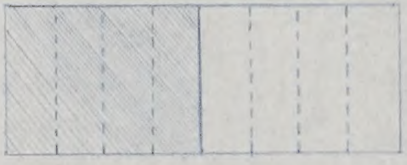
Grades 6 and 7

December

1. Show 5/6 of this pie by coloring it in.



2. Into how many fourths can I cut a pie?
3. Which is larger, 2 1/4 or 8/4?
4.



If I eat the shaded part of this chocolate bar, what part will be left to eat tomorrow?

5. Which is larger, 1/2 or 5/8?
6. 1 1/3 + 1 2/3 + 2 1/3
7. 5/8 + 1/8 + 7/8
8. 3 1/2 + 4 1/2 + 2 1/2
9. 3/4 + 3/4
10. 1/2 + 3/4 + 1/2
11. 3 1/3 - 2 2/3
12. 8 - 6 3/4
13. 4 1/4 - 2
14. 6 3/4 - 2 1/2
15. 7/8 - 3/8
16. 2/3 of 9
17. 3/4 of 16
18. 1/4 of 1/2

19. \$.40
x 3 3/8

20. \$.18
x 5 1/2

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